OCR
RECOGNISING ACHIEVEMENT

OXFORD CAMBRIDGE AND RSA EXAMINATIONS

Advanced Subsidiary General Certificate of Education
Advanced General Certificate of Education

MEI STRUCTURED MATHEMATICS 2613

Statistics 1

Tuesday **28 May 2002** Afternoon 1 hour 20 minutes

Additional materials:
Answer paper
Graph paper
MEI Examination Formulae and Tables (MF12)

TIME 1 hour 20 minutes

INSTRUCTIONS TO CANDIDATES

- Write your Name, Centre Number and Candidate Number in the spaces provided on the answer booklet.
- Answer **all** questions.
- You are permitted to use a graphical calculator in this paper.

INFORMATION FOR CANDIDATES

- The approximate allocation of marks is given in brackets [ ] at the end of each question or part question.
- You are advised that an answer may receive no marks unless sufficient detail of the working is shown to indicate that a correct method is being used.
- Final answers should be given to a degree of accuracy appropriate to the context.
- The total number of marks for this paper is 60.

This question paper consists of 5 printed pages and 3 blank pages.

1 The magazine *Nearly Eighteen* has a web site, on which it recently ran a pop trivia quiz with ten questions. The results of the first 1000 entries were analysed. The numbers of questions answered correctly, x, are illustrated by the frequency diagram in Fig. 1.

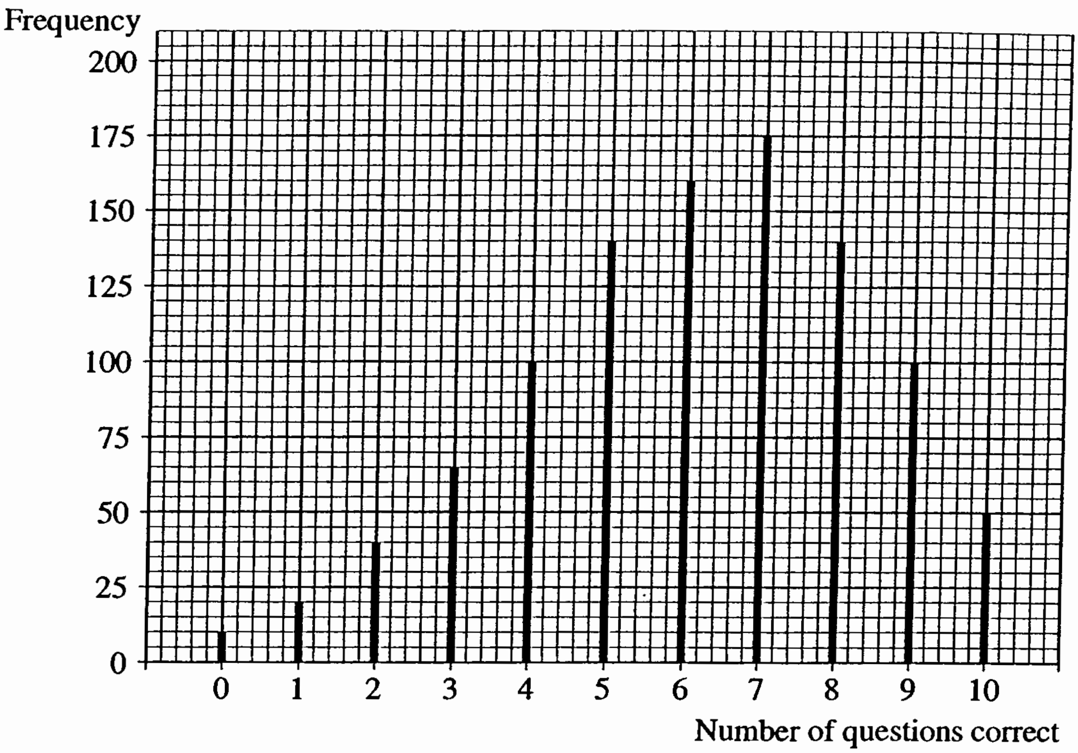

Fig. 1

$$n = 1000 \qquad \sum fx = 6100 \qquad \sum fx^2 = 42\,260$$

(i) Find the mode and the median of the data, and describe briefly the shape of the distribution. [4]

(ii) Illustrate the data using a box and whisker plot. [3]

(iii) Calculate the mean and standard deviation of the data. [3]

Each question in the quiz was in fact of the multiple choice variety, with four possible answers. Three points are awarded for a question answered correctly, and one point is deducted for a question which is not answered correctly.

(iv) Show that, if x questions are answered correctly, the number of points, y, is given by

$$y = 4x - 10.$$ [2]

(v) Hence find the mean and standard deviation of the number of points scored. [3]

[Total 15]

2 Shaldon High School has two AS mathematics classes, called East and West. East and West each have 15 boys and 5 girls.

The East class chooses 3 students at random to represent it on the MCC (Mathematics Consultative Committee).

(i) Copy and complete the probability tree diagram in Fig. 2 to show all the possible choices of boys and girls from the East class. [3]

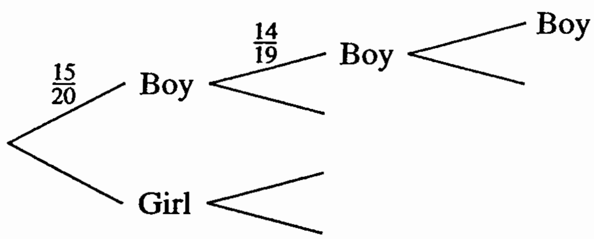

Fig. 2

(ii) Find the probability that the East class is represented by

(A) three boys, [2]

(B) two boys and one girl. [3]

The West class also chooses 3 students at random to represent it on the MCC, so that the MCC has 6 students altogether.

(iii) Find the probability that the MCC contains

(A) one girl and five boys, [3]

(B) at most one girl. [2]

(iv) Given that the MCC contains *at most* one girl, find the probability that it contains *exactly* one girl. [2]

[Total 15]

[Turn over

3 Delightful Desserts produces pots of yoghurt in five flavours with the following proportions.

Black Cherry	30%
Peach	20%
Raspberry	20%
Rhubarb	10%
Strawberry	20%

For each flavour, an average of 5% of pots fail a taste test.

During each shift, the quality control department samples 50 pots of yoghurt for testing purposes.

(i) Give two distinct reasons why taking 50 pots from the first hour's production is an inappropriate method of sampling. [2]

(ii) Describe a suitable method of selecting the sample of 50 pots, stating the number of pots of each flavour that should be in the sample. [4]

All the raspberry yoghurts produced during a shift are rejected if a random sample of 10 pots contains more than one pot which fails the taste test.

(iii) Find the probability that

(*A*) no raspberry yoghurts fail the test, [2]

(*B*) just one raspberry yoghurt fails the test, [3]

(*C*) all the raspberry yoghurts are rejected. [3]

[Total 14]

4 A police road-safety team examines the tyres of a large number of commercial vehicles. They find that 17% of lorries and 20% of vans have defective tyres.

(i) Six lorries are stopped at random by the road-safety team. Find the probability that

(*A*) none of the lorries has defective tyres, [2]

(*B*) exactly two lorries have defective tyres. [3]

Following a road-safety campaign to reduce the proportion of vehicles with defective tyres, 18 vans are stopped at random and their tyres are inspected. Just one of the vans has defective tyres.

You are to carry out a suitable hypothesis test to examine whether the campaign appears to have been successful.

(ii) State your hypotheses clearly, justifying the form of the alternative hypothesis. [4]

(iii) Carry out the test at the 5% significance level, stating your conclusions clearly. [3]

(iv) State, with a reason, the critical value for the test. [2]

(v) Give a level of significance such that you would come to the opposite conclusion for your test. Explain your reasoning. [2]

[Total 16]

Mark Scheme

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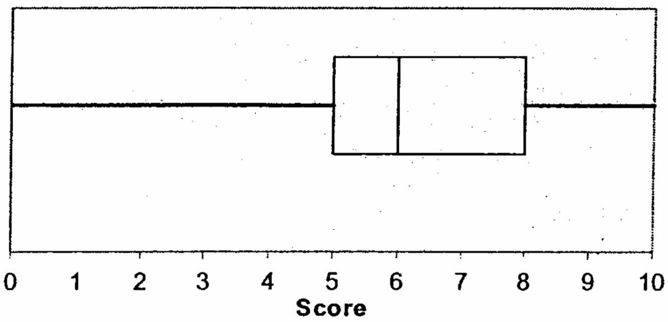
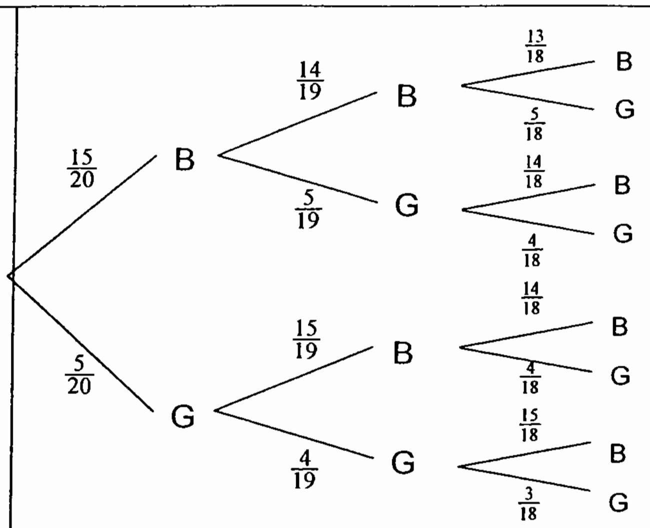
Question 1

(i)	Mode = 7 (most frequent score) 375 scores ≤ 5, but 535 scores ≤ 6, hence Median = 6 Distribution exhibits negative skewness *or* is unimodal	B1 M1 using cum. freqs. A1 B1	4
(ii)		G1 for annotated structure with range 0 to 10 G1 for whiskers from 0 to 5 and 8 to 10 G1 for box from 5 to 8 and median **at 6**	3
(iii)	Mean $= \frac{6100}{1000} = 6.1$ s.d. $= \sqrt{\frac{42260}{1000} - 6.1^2} = \sqrt{5.05} = 2.25$ (3 s.f.) *or* 2.2 (2 s.f.)	B1 for mean M1 for variance A1 **cao** (inc. s.d.)	3
(iv)	Number of points $= 3x - (10 - x) = 4x - 10$	B1 for "$(10 - x)$" B1 for "$3x - \ldots$"	2
(v)	$\bar{y} = 4\bar{x} - 10 = 4 \times 6.1 - 10 = 14.4$ s.d.$(y) = 4 \times$ s.d.(x) $= 4 \times 2.247\ldots = 8.99$ (to 3 s.f.) or 9.0 (to 2 s.f.)	M1 for $\bar{y}$ A1 **cao** B1 for s.d.(y)	3
			15

Question 2

(i)		G1 for 1st set [both correct] G1 for 2nd set [all 4 correct] G1 for 3rd set [all 8 correct]	**3**
(ii)	(*A*) P(East class represented by 3 boys) $= \frac{15}{20}\times\frac{14}{19}\times\frac{13}{18} = \frac{91}{228}$ [*or* ${}^{15}C_3 \div {}^{20}C_3$] $= 0.399$ (to 3 s.f.) $= 0.4(0)$ (2 s.f.) (*B*) P(East class represented by 2 boys & 1 girl) $= \frac{15}{20}\times\frac{14}{19}\times\frac{5}{18} + \frac{15}{20}\times\frac{5}{19}\times\frac{14}{18} + \frac{5}{20}\times\frac{15}{19}\times\frac{14}{18}$ $= \frac{35}{76} = 0.46$ (to 2 s.f.) [*or* ${}^{15}C_2 \times {}^{5}C_1 \div {}^{20}C_3$]	M1 for product of 3 probabilities *or* ratio of ${}^nC_r$ terms A1 **cao** M1 for $\geq$ 1 triple product M1 for sum of 3 triple products A1 **cao** [*or* M1 top, M1 bottom]	**5**
(iii)	(*A*) P(1 girl and 5 boys on the MCC) = P(3 boys from East and 2 boys, 1 girl from West) + P(2 boys, 1 girl from East and 3 boys from West) $= \frac{91}{228}\times\frac{35}{76} + \frac{35}{76}\times\frac{91}{228} = \frac{3185}{8664} = 0.3676 = 0.37$ (to 2 s.f.) [*or* $\left(\frac{15}{20}\times\frac{14}{19}\times\frac{13}{18}\times\frac{15}{20}\times\frac{14}{19}\times\frac{5}{18}\right)\times 6$] (*B*) P(at most 1 girl on the MCC) = P(0 or 1 girls on the MCC) $= \frac{3185}{8664} + \left(\frac{91}{228}\right)^2 = 0.53$ (to 2 s.f.)	M1 for 1 product M1 for sum of 2 prods. A1 **cao** [*or* M1 sextuple product M1 "$\times 6$"] M1 for sum of 2 prods. A1 **cao**	**5**
(iv)	Conditional probability $= \dfrac{0.3676...}{0.5269...}$ [= 0.70 (to 2 s.f.)]	M1 for denominator M1 for numerator [dep]	**2**
			15

Question 3

(i)	The sample could well be biased, since the yoghurts chosen might not be representative of flavour or those produced throughout the shift.	E1 for ref. to flavours E1 for ref. to time	2
(ii)	Stratified *or* quota *or* systematic sampling where the 50 yoghurts are sampled throughout the shift in proportion to the production ratios. $\Rightarrow$ Black Cherry 15, Peach 10 Raspberry 10, Rhubarb 5 Strawberry 10	B1 for stratified *or* quota *or* systematic B1 representative of time M1 for flavours in proportion A1 for sample numbers	4
(iii)	(*A*) P(no raspberry yoghurts fail the taste test) $= 0.95^{10} = 0.599$ (to 3 s.f.) $= 0.6$ (to 1 s.f.) *or from tables*: 0.5987	M1 A1 **cao**	2
	(*B*) P(just one raspberry yoghurt fail the taste test) $= 10 \times 0.95^9 \times 0.05 = 0.315$ (3 s.f.) $= 0.32$ (2 s.f.) *or from tables*: $0.9139 - 0.5987 = 0.3152$	M1 for "0.05" (SOI) M1 for binomial term A1 **cao**	3
	(*C*) P(all raspberry yoghurts rejected) $= 1 - \text{P(0 or 1 yoghurt fails the test)}$ $= 1 - (0.3152 + 0.5987)$ $= 1 - 0.9139$ $= 0.086$ (to 3 s.f.) *or from tables*: $1 - 0.9139 = 0.0861$	M1 for their (A) + their (B) M1 for $1 - [\text{P}(0) + \text{P}(1)]$ A1 **cao**	3
			14

Question 4

(i)	(*A*) P(no lorries have defective tyres) $= 0.83^6 = 0.327$ (3 s.f.) $= 0.33$ (2 s.f.) (*B*) P(exactly 2 lorries have defective tyres) $= {}^6C_2 \times 0.17^2 \times 0.83^4$ $= 0.206$ (to 3 s.f.) $= 0.21$ (2 s.f.)	M1 A1 **cao** M1 for $0.17^2 \times 0.83^4$ M1 for ${}^6C_2 \times \ldots$ A1 **cao**	**5**
(ii)	H_0: $p = 0.2$ H_1: $p < 0.2$ H_1 takes this form because we are looking for a *reduction* in the proportion of defective tyres.	B1 for null hypothesis B1 for alternative hyp. E1 for explanation	**3**
(iii)	Let $X \sim B(18, 0.2)$ $P(X \leq 1) = 0.0991$ Since $0.0991 > 0.05$, do not reject H_0 (*or* accept H_0) There is not enough evidence to suggest that there has been a (significant) reduction in the proportion of defective tyres *or* "campaign appears to have been successful"	B1 for tail probability M1 for comparison M1 for "accept H_0" [dep.] A1 for conclusion in words	**4**
(iv)	The critical value for the test is 0, since $P(X \leq 0)$ [$= 0.018$] < 0.05	B1 for critical value E1 for reason	**2**
(v)	The opposite conclusion would be reached provided the significance level was above 9.91%, e.g. 10%	B1 for suitable percentage E1 for explicit comparison with 9.91%	**2**
			16

Examiner’s Report

2613 Statistics 1

General Comments

There were more higher marks than usual and fewer very poor scripts. In many cases centres had prepared their candidates well for the examination.

Mistakes where they did arise consisted of curious formulae for the standard deviation, adding probabilities instead of multiplying or vice versa, squaring instead of doubling or vice versa, omitting ^nC_r terms where needed, performing " 1 –….." when they should not have, omitting the parameter p from the hypotheses and using point probabilities instead of tail probabilities.

Comments on Individual Questions

Q.1 Candidates generally performed very well on question 1, most of which was accessible to all candidates. However, the linear coding section proved to be quite challenging, even to the more able candidates.

(i) The mode (7) and median (6) were usually found correctly, with 6 sometimes appearing via $\frac{1}{2}$ (11 + 1). Some candidates made errors in the cumulative totals leading to 5 or 9. A few tried to interpolate between 5 and 6 getting a median of 5 point something. Many recognised that the shape of the curve exhibited negative skewness but there are still too many candidates who believe that "right skew" is an acceptable alternative, which it is not.

(ii) Those who failed to find the median usually failed similarly with the quartiles. 4 and 7 or 9 were the usual alternatives or interpolation again and, in rare cases, trying to use a cumulative frequency diagram. Several did not extend the left whisker to zero. Whilst many knew the structure of a box and whisker diagram many inserted the mode or even the mean inside the box.

(iii) The mean and standard deviation were usually calculated correctly. Some ignored the given Σfx and Σfx^2 and attempted to find them from the graph.

(iv) Very few candidates did the expected $3x - (10 - x)$. Most verified the result by doing so many 3's and so many –1's, then checking that $4x - 10$ gave the same answer. Often, however, only one value of x was checked. An alternative solution was to find the equation of the line through e.g. (0, –10) and (10,30) using $y = mx + c$ or $y - k = m(x - h)$. There were several successes this way. Some weaker candidates believed the 4 in "$4x - 10$" was "because there are 4 possible answers".

(v) Few candidates were familiar with $\bar{y} = 4\bar{x} - 10$ and even fewer with s.d.(y) = 4 s.d.(x). Standard deviation = –1.01 was seen several times. Many, unfortunately, recalculated using the y-values. Often the frequency was omitted and/or errors were made in the ensuing arithmetic.

(i) mode = 7, median = 6, negative skew; (ii) box and whisker diagram;
(iii) mean = 6.1, s.d. = 2.25; (iv) $3x - (10 - x)$; (v) mean = 14.4, s.d. = 8.99

Q.2 In question 2, the majority of candidates completed the tree diagram and early probability calculations with little difficulty. Predictably, answers to the later parts varied considerably across the entry.

(i) Using the hint in the question, a large majority of candidates produced a tree diagram with correct probabilities. Those who completed the tree diagram correctly usually scored well in part (ii).

(ii) Part (A) was usually completed correctly. Part (B) was also well done, but some only found two triplets, or multiplied instead of adding. Premature approximation was also rife. Each triple product was 0.1535. This was often rounded to 0.15 followed by 0.15 + 0 .15 + 0.15 = 0.45(instead of 0.46). **Errors of this type were common throughout the paper leading to the loss of valuable marks.**

(iii) In contrast to part (ii), attempts at solutions were generally disappointing. Many candidates considered choosing 6 from 40 instead of 3 from 20 twice. Probably, many did not realise that it would not give the same answers; they thought they had found an easier way of doing it. Those using the correct method were often successful and many scored the 5 marks available. However, adding instead of multiplying, or forgetting to multiply by 2 were often seen. In fact, $\frac{91}{228}+\frac{35}{76}=\frac{49}{57}$ was a frequent wrong answer for part (A). In part (B) $\left(\frac{91}{228}\right)\times 2$, instead of $\left(\frac{91}{228}\right)^2$, was seen fairly often.

(iv) Those who reached the end of the question were often able to score the two marks for $\frac{\text{their (iii) A}}{\text{their (iii) B}}$, even if they had made a complete mess of part (iii). Many who had used the "40 method" got the two marks here on a follow-through.

(i) tree diagram; (ii) (*A*) $\frac{91}{228}=0.40$, (*B*) $\frac{35}{76}=0.46$; (iii) (*A*) $\frac{3185}{8664}=0.37$, (*B*) 0.53;
(iv) 0.70 (to 2 s.f.).

Q.3 Most candidates made a good start to question 3 by writing sensible responses in the sampling section. A large number also scored well on the probability section which followed, except for the last part where the logic of the situation often defeated them.

(i) Many naïve comments were made about workers being half-asleep or being hungover, or machinery not being warmed up yet, or ingredients left over from yesterday and such nonsense. However, most candidates managed to talk enough sense to achieve at least one mark, often after taking half a side to say, " an hour isn't long enough".

(ii) Nearly all candidates named a suitable sampling method and produced the correct sampling numbers. Very few mentioned that it should take longer than an hour to be an improvement on the first method.

(iii) Despite being told that, "For each flavour 5% fail", many felt that some other failure rate applied to the raspberry yoghurt; 0.5%, 1%, 10%, 20% and 50% were all used instead of 5%. Where the correct probability was used parts (A) and (B) were nearly always correct. However, the answers to part (C) was nearly always incorrect. There was some confusion between " More than one" and " One or more", but most wrongly found P(all to fail) $= 9.8\times 10^{-14}$. As one candidate put it, "very unlikely".

(i) comment on unsatisfactory sampling; (ii) stratified *or* quota sample: 15, 10, 10, 5, 10;
(iii) (*A*) 0.599, (*B*) 0.315, (*C*) 0.0861 (to 3 s.f.).

Q.4 Responses to question 4 were variable; most candidates handled the probability calculations reasonably well, but once again the hypothesis testing section provided responses ranging from being totally correct to utter nonsense.

(i) This part was done well. Many scored the full 5 marks here. The main reasons for failure were using the wrong value for *p*, 0.20 instead of 0.17, or omitting the $^6C_2$ in part (B).

(ii) The hypotheses were mainly stated correctly, albeit in words in many cases. However, many explanations for the form of the alternative hypothesis were incorrect. Many said, "because only 1 out of 18 had defective tyres". Others did not give a reason, but merely re-stated the alternative hypothesis in words.

(iii) The carrying out of the hypothesis test was often well done. Most got 0.0991 > 0.05, therefore accept H_0. Conclusions in plain English were sometimes omitted. Many candidates found the critical region first, then proceeded to explain why 1 was not in the critical region. This approach gained full credit. There were a few sights of "$\frac{1}{18}$ = 5.6%, 5.6% > 5%". A significant minority used point probabilities which, as is customary, gained no credit.

(iv) A pleasing number recognised 0 as the critical value. '1' or '$\geqslant 1$' was a frequent alternative.

(v) 10% was frequently seen, sometimes with the correct reason, but more often without.

(i) (*A*) 0.33, (*B*) 0.21 (to 3 s.f.); (ii) $H_0: p = 0.2$, $H_1: p < 0.2$, comment;
(iii) $P(X \leqslant 1) = 0.0991 > 0.05$, hence accept H_0; (iv) critical value = 0; (v) e.g. 10%